

Multiple Representations in Preschool Numeracy: Teaching a Lesson on More-or-Less

Alfredo Bautista¹⁾

The Education University of Hong Kong

Malikka Habib

Nanyang Technological University

Raymond Ong

Nanyang Technological University

Anthony Eng

Rebecca Bull

Macquarie University

Abstract

Mathematics education researchers and curriculum designers have emphasized the importance of representations in fostering students' mathematical learning. However, most classroom-based studies on representations have focused on primary and secondary levels, and there is limited literature on how preschool teachers integrate representations in regular classroom activities. This qualitative study illustrates specific ways in which preschool educators may utilize multiple representations to teach numeracy, thereby showing how theoretical ideas can be applied into practice. We analyze a lesson intended to help 4-5 year old children learn about the concept of more-or-less and about the process of determining differences between two quantities. After describing the structure of the lesson, we analyze the activities and pedagogical strategies employed by the teacher to achieve the intended learning objectives, with special emphasis on the mediation of representations as learning tools. Our ultimate goal is to encourage preschool educators to utilize multiple representations strategically within the context of whole-class and small-group activities.

Keywords: curriculum, pedagogy, numeracy, multiple representations

Corresponding author, ¹⁾ abautista@eduhk.hk

Introduction

Early childhood education researchers have underlined the need to strengthen preschool teachers' pedagogical approaches to early mathematics (Alsina, 2013; Moon & Lee, 2011). This study focuses on the topic of representations as teaching and learning tools (Brizuela & Gravel, 2013). While students' use of representations in mathematics is a well-established field of research, few classroom-based studies have documented how preschool educators may utilize representations to teach about numeracy notions and processes to young children. To enrich the limited pedagogical literature on the use of multiple representations at the preschool level, we analyze a lesson that illustrates how teachers can apply theoretical ideas into whole-class and small-group learning activities. Our ultimate goal is to increase awareness among preschool educators about the important role representations play in numeracy learning, and provide insights that inspire them to utilize multiple representations in their own lessons.

Literature Review

Mathematical representations and their role in preschool curriculum frameworks

Mathematics education researchers define the notion of *representation* as any sign, character, and object that stands for something other than itself, and that is used to symbolize, describe, depict, encode, and/or represent mathematical concepts and processes (Goldin & Shteingold, 2001). Some of the more common types of representations used for the teaching and learning of school mathematics are concrete (including manipulatives), embodied (gestures), pictorial (drawings), graphical (models, graphs), verbal (words, statements), tabular, as well as numerical and other symbolic representations (Dreher, Kuntze, & Lerman, 2016; Kamii, Lewis, & Kirkland, 2001; Sinclair & Pimm, 2015). Each representation highlights specific aspects or properties of a given mathematical notion while obscuring others. For this reason, researchers have argued that using multiple representations that complement one another and analyzing the connections among them are powerful strategies for learners to develop a more complete and meaningful understanding of mathematics (Goldin & Kaput, 1996).

There is a large body of research on young children's use of mathematical representations. Goldin and Kaput (1996) established the characteristics of different types of representations (e.g., internal vs external, conventional vs invented), analyzed their roles in the learning of mathematics, and identified three stages in children's development of new representations: "(a) an inventive-semiotic stage, (b) a period of structural development, and (c) an autonomous stage" (p. 424). Subsequently, other researchers have investigated children's own creation and use of representations within classroom settings, including mathematical scribble-marks (Carruthers & Worthington, 2005), graphics (Carruthers & Worthington, 2008), and drawings and narratives (MacDonald, 2013). This body of work indicates that representations are not only an important window into how children construct meaningful mathematical understandings, but the very foundation of early mathematics.

Intervention studies have provided empirical evidence for the positive effects that learning through multiple representations has on children's number sense and subsequent mathematical understanding. For example, Gürbüz (2010) showed the potential of multiple representations in helping Grade 7 students (11-12 years old) to recognize the common mathematical elements of different situations. Children in classrooms where the teacher incorporated manipulatives seemed to identify such elements significantly better than those children whose teachers did not use manipulatives. This study concluded that because representations make mathematical ideas more concrete and available for reflection, they are powerful tools for building mathematical understanding, communicating information, and demonstrating reasoning. Sherman and Bisanz (2009) conducted a study with Grade 2 students (7-8 years old) who had difficulties solving equivalence problems (e.g., $5 + 4 = 7 + ?$). The authors developed a non-symbolic method, using blocks, to represent equivalences. Children performed significantly better when using the non-symbolic method, which suggested that their struggles were actually due to lack of understanding of symbolic arithmetic notation (i.e., the + and = signs) rather than lack of understanding of the notion of equivalence itself.

The importance of representations has been also emphasized in contemporary early childhood curriculum frameworks around the world. Consistent with recommendations by researchers (Carbonneau, Marley, & Selig, 2013), there seems to be clear recognition

among curriculum designers that adopting a multiple representation approach significantly enhances young children's numeracy learning. In the United States, for example, preschool educators are encouraged to teach about numbers using sets of concrete manipulatives, as well as pictorial and graphical representations (Common Core State Standards Initiative, 2010). The National Kindergarten Curriculum from South Korea suggests that children's development of number sense should rely on both concrete and abstract representations, and that children need to actively interact with such representations to learn how to solve every-day problems independently (Ministry of Education and Human Resources Development, 2007). Similar ideas can also be observed in Nurturing Early Learners (NEL), the national kindergarten curriculum framework in Singapore (Ministry of Education [MOE], 2013a). The NEL is a national guideline designed by the Ministry of Education to raise teaching quality standards and harmonize pedagogical practices across the Singapore preschool sector (for detailed information about Singapore's preschool education system, see Bull & Bautista, 2018).

In the NEL, *Numeracy* is considered as one of the key learning areas for children's holistic development. The *Numeracy* curriculum guide (MOE, 2013b) places much emphasis on representations as teaching and learning tools, and suggests numerous activities in which representations play a central role. For example, one activity proposes giving a bag of counters and a paper plate to each child, pairing children up, asking them to place some counters on their plates and compare who has more. Verbalizing numbers, forming them with play dough, indicating them with fingers, writing them in conventional numerals or number words, or visualizing them on a number line are proposed as instructional strategies to foster children's number sense. The view is that for the concept of number to be meaningful, learning activities must be engaging, authentic, and connected to the real world. Furthermore, the NEL proposes that children's numeracy learning can occur throughout the day, being embedded in daily routines, and play settings and hands-on activities planned by the teacher, ranging from individual to small and large group activities (MOE, 2013b).

The NEL suggests the Concrete-Pictorial-Abstract (CPA) approach as a guideline to teach numeracy to young children (Leong, Ho, & Cheng, 2015). The CPA approach is based on the instructional theories proposed by Bruner (1966), particularly on his

conception of the *enactive*, *iconic*, and *symbolic* modes of representation. The understanding is that learning should start from embodied hands-on experiences with physical objects and manipulatives, for example, counting actual apples or replica toy apples (i.e., enactive representations). Once children demonstrate understanding at the concrete level, teachers would introduce pictorial representations of those objects or manipulatives, for example, counting dots that stand for apples on a piece of paper or on the board (i.e., iconic representations). Finally, once the second phase is mastered, teachers would progressively introduce more abstract ways of representation such as numerals or number words (i.e., symbolic representations). With adequate scaffolding, children would come to understand that those arbitrary symbols are valid ways to represent numbers, either the number of concrete objects (e.g., apples) or the number of pictorial objects (e.g., dots that represent apples).

The role of representations in the classroom: Teachers' beliefs and practices

Few observational studies have explored how preschool educators integrate representations in regular classroom activities, as well as the pedagogical strategies adopted to scaffold children's understanding of such representations. Braswell (2015) examined how different types of representations, spaces, and adult-mediated routines shaped children's representational development. Conducted with 4- to 5-year-old children, the study explored how teachers' mediation strategies influenced children's understandings and productions of numeracy and literacy external representations. Three types of mediation strategies were identified: explicit mediation (including prompting, describing processes, and giving feedback to children about their use of representations), implicit mediation (showing children how to create a representation via imitation and modeling), and acting as an equal to children (serving as a play partner). Moreover, preschool teachers were observed helping children to link different representations using a range of connection-making strategies, for example, using artifacts across contexts (e.g., using cookies at snack time to count) and relating classroom activities with activities that happen outside of the classroom.

Delacour (2016) investigated how two preschool educators encouraged Swedish children to learn numeracy concepts using physical objects and materials (e.g., metal cans, jars) in

an outdoor environment. The focus was to analyze how teachers' communication strategies could affect children's learning. Both teachers were observed introducing and explaining mathematical notions while commenting on the properties of the items used to complete the problem at hand. The teachers repeated, extended, clarified, and asked further questions to allow children to discover that different solutions could be equally valid to solve the same problem. The study provides insights into how preschool teachers can reinforce children's numeracy learning by posing engaging problems and allowing free exploration of the physical environment.

Most studies focusing on how teachers utilize representations have been conducted in primary/elementary and secondary/middle-school classrooms. A case study by Jao (2009) examined the strategies used by an elementary teacher (Sabrina) to scaffold the construction of her students' mathematical knowledge via multiple representations. When teaching a new concept, Sabrina first asked her learners what they already knew about it, and then asked them to represent the concept in any way possible (e.g., using words, pictures, or numbers), and to know more about what representational forms they were inclined to use naturally. In her lessons, she facilitated students' progression from concrete to abstract forms of presentation. During individual interviews, Sabrina stressed the importance of being able to use and understand many types of representations. She believed that being able to *maneuver* between representational formats ensures a true understanding of mathematics. She expressed that when only one type of representation is used, students could be simply memorizing without understanding (e.g., learning a rote formula).

Findings from studies of professional development (PD) have also highlighted the need for further guidance in the ways teachers integrate representations in their teaching. Puchner, Taylor, O'Donnell, and Fick (2008) analyzed the use of manipulatives in lessons taught by four groups of elementary mathematics teachers (K-8) involved in a lesson study project as part of their PD. It was found that in three of four lessons analyzed, the use of manipulatives was turned into an end in and of itself rather than a learning tool. Further still, in the fourth lesson, the use of manipulatives hindered rather than helped student learning. These surprising findings were interpreted as a consequence of teachers' inadequate pedagogical use of manipulatives. Manipulatives were not particularly helpful due to the absence of a challenging enough problem. Students used manipulatives to obtain answers

they were able to obtain easily using traditional algorithms. This study provided valuable guidance for planning of future PD, stressing the need to help mathematics teachers understand how and when representations should be integrated to foster students' mathematical learning more successfully.

Moreover, findings from PD studies have shown that PD can positively enhance teachers' pedagogical strategies pertaining to the use of representations. Bautista, Cañadas, Brizuela, and Schliemann (2015) examined the impact of an intensive, content-focused PD on grades 5-9 mathematics teachers' pedagogical use of representations, with a particular focus on the Cartesian graph. The use of graphs was infrequent prior to the PD, particularly in grades 5-6. When graphs were used, these were rarely used in conjunction with other forms of representation. However, towards the end of the PD, teachers from all grade levels started to utilize multiple representations more frequently and were successful in establishing connections between graphs and other forms of representations. Findings suggested that the specific features of the PD program contributed to enhance teachers' pedagogical use of graphs.

Teachers' infrequent use and difficulties with representations have been attributed to factors such as limited mathematical content knowledge, lack of pedagogical preparation, misconceptions about the function of representation, as well as negative views and beliefs about their importance in mathematics education. Dreher and Kuntze (2015) found that neither pre-service nor in-service German middle school teachers believed that representations supported students' understanding or remembering of mathematical concepts, nor motivated and interested students. In another study, Dreher et al. (2016) examined the views about the role of representations in German and English pre-service teachers. While certain culture-dependent aspects were identified, findings indicated that neither English nor German teachers sufficiently valued the importance of representations. Similar results were obtained by Stylianou (2010), who interviewed middle school mathematics teachers to explore their conceptions of representations. It was found that representations were viewed by some teachers as a *topic* of study rather than as a *tool* for mathematics teaching and learning. Certain teachers expressed that having multiple representations might cause confusions in students, especially in low-achievers, and suggested that only high-performing students should be introduced to this topic.

The Present Study

In line with the studies described above, early childhood education researchers have also underlined the need to strengthen preschool teachers' pedagogical approaches to early mathematics, as important discrepancies seem to exist between curriculum frameworks and practices on the ground (Alsina, 2013; Moon & Lee, 2011). This paper responds to calls for situated examples of good classroom practices such as the call by Darling-Hammond and McLaughlin (2011), who argued that to enhance teacher quality and ultimately student learning, "teachers must learn about, see, and experience successful learning-centered and learner-centered teaching practices" (p. 83).

In addition, there is scarcity of research focusing on how preschool educators utilize representations to facilitate young children's numeracy learning. Existing studies have centered exclusively on the use of manipulatives in outdoor environments (Delacour, 2016), and the interactions between representations, spaces, and teachers' classroom routines in numeracy and literacy (Braswell, 2015). In contrast, the present qualitative study illustrates specific ways in which preschool educators may integrate different types of representations in regular classroom activities, including whole-class and small-group activities, and analyzes pedagogical strategies that teachers may adopt to scaffold children's understanding of such representations. We aim to show how theoretical ideas can be applied into practice, with the ultimate goal of encouraging other preschool educators to integrate multiple representations in their own lessons.

Method

Context, Participants, and Case Selection

The sample case analyzed in this study was identified in the context of the Singapore Kindergarten Impact Project (SKIP), a large-scale longitudinal study examining how the preschool environment and pedagogical practices, together with home factors, influenced children's learning and development in Singapore. SKIP involved 1,538 Kindergarten 1

(K1) children (4-5 years old) from 80 kindergarten and child care centers from a range of social strata, geographical locations, and types of provider (both public and private). Classroom observations were conducted to measure program quality with regard to structural and process factors.

A total of 113 K1 teachers were observed. We identified instances related to *Numeracy* in only 27 of these teachers (24%). The sample case presented here was selected using an information-oriented sampling strategy (Sandelowski, 1995), that is, due to the intrinsic interest of the activities and pedagogical strategies utilized by the teacher, who displayed the most outstanding use of multiple representations. Note that teachers' usage of mathematical representations was not the main focus of SKIP. However, upon viewing the data, it became clear that the information collected from one of the participants could illuminate our understanding of this topic. As such, the current study should be interpreted as a secondary data analysis.

The teacher featured in the paper, referred to as Miss Smith (pseudonym), was 34 years old and had five years of preschool teaching experience. She held a Diploma in Early Childhood Education, which requires the equivalent of 18 months full-time study and approximately 300 practicum hours. Her class consisted of 17 preschool children (8 boys and 9 girls). The lesson was recorded in July 2015.

Procedure

Two research assistants observed and videotaped each classroom for 3-4 hours (depending on the duration of the program). The person operating the video camera followed the teachers at all times in order to capture their interactions with children. Because our interest was to capture instances of the regular operations within the participating preschools, we explained to both center principals and teachers that our observations should be conducted during a "typical" teaching and learning day. Teachers were given no instructions or directions regarding the content or the pedagogy of the activities to be implemented that day.

Ethics approval was obtained from the authors' university Institutional Review Board (IRB). Once a preschool center leader had indicated their willingness to participate in SKIP,

teachers were asked to indicate whether they consented to participate. Parental consent forms were sent to the parents of children whose teacher consented to participate. Only children whose parents provided written consent in the first wave of data collection were included as SKIP participants.

Findings

Lesson Objective and Structure

Miss Smith implemented a lesson intended to help 4- to 5-year-old children learn about the concept of more-or-less and about the process of determining differences between two quantities. The lesson was structured in the following three segments.

- **Assembly Time (7 minutes):** During this time, children sang nursery rhymes related to numeracy concepts. Miss Smith gestured to the rhymes along with the children.
- **Large Group Activities (13 minutes):** Miss Smith implemented three whole-class activities involving the use of multiple representations, which were carried out sequentially:
 - Activity 1.* Spiders and ants: Who have more legs?
 - Activity 2.* Which hula-hoop has more children?
 - Activity 3.* Which tray has more penguins?
- **Pair Working Stations (15 minutes):** Miss Smith asked children to play in the various learning corners available in the classroom (e.g., dramatic play, reading corner). She brought a group of six children to the numeracy corner and divided them into three pairs. There were three working stations set up in a round table, each with different materials. The teacher asked each pair to work for five minutes in each of the stations:
 - Station A.* Die, counting trays, and penguin counters.
 - Station B.* Dot cards.
 - Station C.* Cards with pictures, numerals, and number words.

Below, we focus on the observed use of representations within large group activities and pair working stations. We present key conversational exchanges between Miss Smith and the children to illustrate how the lesson unfolded. All children's names are pseudonyms. In

addition to transcripts, we present images of the representations utilized (e.g., screenshots, diagrams), describe relevant bodily gestures performed by the teachers and the children, and occasionally provide information about their voice intonation (for example, to indicate enthusiasm).

Use of Representations in Large Group Activities

Once assembly time was over, Miss Smith implemented three whole-class activities. All the children were seated together in front of her.

Activity 1. Spiders and ants: Who have more legs?

Miss Smith started by asking children about the number of legs that spiders and ants have. Children knew the correct answer, as this topic had been discussed in the previous day, and quickly responded eight and six verbally. The teacher acknowledged children's responses and showed them the piece of mahjong paper used during the prior class (see Figure 1). It contained a table with two columns: the Spider column, with eight black tally marks, and the Ant column, with six red tally marks.

Miss Smith: What did we mark here? What is this?

[pointing to the black tally marks in the Spider column]

Class: Legs! *[emphatically]*

Miss Smith: Eight legs, very good. And what about this?

[pointing to the red tally marks in the Ants column]

Class: Six Legs!

Miss Smith: Six legs. Sure! Let's count again!

Miss Smith started to count the number of legs in the Spider column with the children, pointing one by one to the black tally marks. "One, two, three, four, five, six, seven, eight!" She then wrote "8 legs" on the top of the Spider column. She repeated the same process for the Ant column, indicating "6 legs" on the top (see Figure 2). She reiterated that spiders have eight legs and ants have six legs.

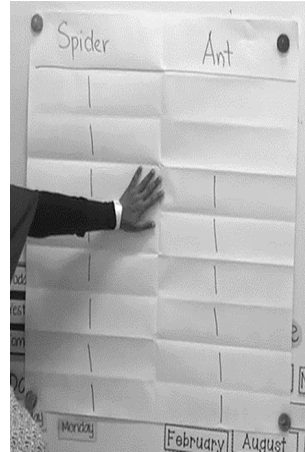


Figure 1. Table Representing Number of Legs with Tally Marks

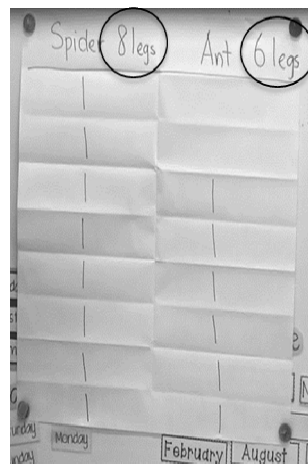


Figure 2. Indicating Number of Legs with Numerals

Miss Smith explained that “Today, we are going to see what is more and less.” Pointing to the two columns on the mahjong paper, she asked the children, “Which are the ones that are the same?” She then started to match the black and red tally marks one by one, drawing lines from the Spider column to the Ant column (Figure 3).

Multiple Representations in Preschool Numeracy: Teaching a Lesson on More-or-Less

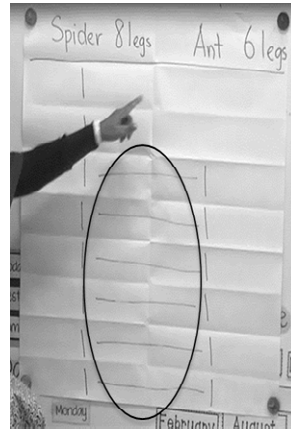


Figure 3. Drawing Lines for Comparison

Miss Smith: The spider has one, the ant has one. So it's the same!

[matching the first pair, starting from the bottom of the paper]

Here, another leg for the spider, another leg for the ant. *[second pair]*

Do we have it again?

Joanne: Yes!

Miss Smith: One for the spider, one for the ant. *[third pair]*

One for the spider, one for the ant. *[fourth pair]*

One for the spider, one for the ant. *[fifth pair]*

One for the spider, one for the ant! *[sixth pair]*

Mark: The ant needs two more!

[pointing to the gap in the Ants column and showing two with his fingers]

Miss Smith: Very good! Now you know! The ants need two more!

Because the ant *[pointing at the Ant column]* has two fewer legs.

Who has more legs?

Several Children: The spider!

Miss Smith: How many more?

Several Children: Eight! Two! *[different responses]*

Miss Smith: Eight? How many more?

[counting the two unmatched black tally marks in the Spider column]

Two! One, two... How many more?

Several Children: Two more!

Miss Smith: How many more?

Class: Two!

Miss Smith: Two more than the ant. The spider has two more legs than the ant. Very good!

Subsequently, the teacher addressed some comments and questions asked by children, using different words to explain the same mathematical idea, namely, that ants have two fewer legs than spiders (e.g., “The ant needs two more legs to make it the same as the spider, or the spider has two more legs than the ant.”).

Activity 2. Which hula-hoop has more children?

Miss Smith began by placing one orange hula-hoop (left) and one yellow hula-hoop (right) on the floor (Figure 4). She then asked two children to stand in the orange hula-hoop and three other children to stand in the yellow hula-hoop (Figure 5). Subsequently, she asked the rest of the class to count the number of people in each hula-hoop. She helped them by counting out-loud while touching the head of each child with her hands. The first objective was to find out which hula-hoop had more children inside.



Figure 4. Use of Two Different Colored Hula-Hoops



Figure 5. Children Standing Inside the Orange (Left) and Yellow (Right) Hula-Hoops

Miss Smith: Which hula-hoop has more?

Class: Yellow *[hand pointing]*

Miss Smith: Yellow, yes! How many more?

Class: Three! Two! *[other inaudible responses]*

While all children were able to identify very quickly that the yellow hula-hoop had more children inside, some had difficulties determining the answer to the question, “How many more?” and gave incorrect responses. To help those children understand, Miss Smith placed her hand on each child while counting the number of children in each hula-hoop. She then matched one child in the yellow hula-hoop to another child in the orange hula-hoop to show that there was one child left with no partner.

Miss Smith: How many more?

Mary: One! *[showing one finger]*

Miss Smith: Look, look. You see? One Mary, one Sarah; the same.

One Jack, one Jill; the same. How many more?

[touching the girl who was left]

Several Children: One more! *[indicating one finger with their hands]*

All children were able to visualize that there was only one child with no match and answered the question correctly. This activity was repeated three times in total, with different number combinations (2-3, 3-1, 1-3). As in the prior activity, the teacher

responded to children's comments and rephrased her own explanations multiple times, using different words.

Activity 3. Which tray has more penguins?

Miss Smith brought a set of objects (Figure 6) comprising of two counting trays (left hand), a large die, one set of white penguin counters, and another set of blue penguin counters. The teacher invited Anthony and Laurie to the front to demonstrate. They were asked to throw the die on the floor, count out loud the number of dots on the resulting upper surface of the die (touching with the finger, one by one), and put the same number of penguins on the counting tray. The task was to compare who had more penguins on his/her tray and determine how many more.



Figure 6. Die, Counting Trays and Penguin Counters

Miss Smith: Okay, now we are going to put it together.

[places both counting trays vertically for comparison, with the penguins side by side; Anthony has three, Laurie has six]

Now, who has more?

Class: Laurie!

Miss Smith: How many more? Ah, that's what we learnt.

Several Children: Four! Three! *[different responses]*

Miss Smith: Listen, okay, let's do it together. One Laurie, one Anthony; same.

[pointing to the first pair, at the bottom]

Multiple Representations in Preschool Numeracy: Teaching a Lesson on More-or-Less

One Laurie, one Anthony; the same. *[second pair]*
One Laurie, one Anthony; the same. *[third pair]*
How many more? *[points to the three counters that are unmatched]*
Class: Three! Three! *[inaudible utterances]*
Miss Smith: Is it three? Let's count it together. *[points to the counter as she counts]*
One, two, three. So, Anthony has...
Several Children: Less.
Miss Smith: Anthony has less. Who has more?
Class: Laurie.
Miss Smith: Laurie, how many more?
Raymond: Anthony needs three.
Miss Smith: Three, yes! *[holds up three fingers]*
Thank you! We will do it again later.

Subsequently, the teacher invited two other children to demonstrate a similar activity in which, instead of throwing a die on the floor, the number of penguins was determined based on the number of dots on paper cards (Figure 7).

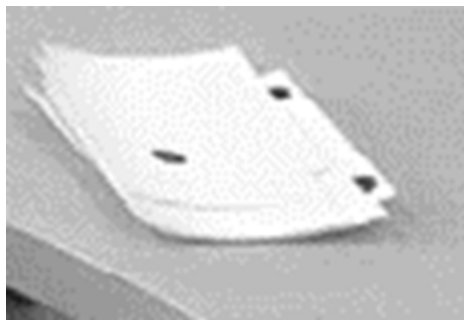


Figure 7. Dot Cards

Use of Representations in Pair Working Stations

Miss Smith brought six children to the numeracy corner and divided them into three pairs. Each pair was asked to work for five minutes in each of the three working stations. During these 15 minutes, the teacher was observed walking around the various learning

corners available in the classroom and talking with all children although she spent more time interacting with the six children in the numeracy stations.

Station A: Die, counting trays, and penguin counters

In this station, the children were provided the same materials used for *Activity 3: Which tray has more penguins?* (see Figure 6). The dynamics were also the same as in *Activity 3*, with the only difference that the pair of children had to work independently (i.e., without the support of the other classmates). The various pairs of children seemed to understand the steps well, and did not demonstrate difficulties in counting the number of dots on the die's upper surface, putting the corresponding number of penguins on the counting tray, or identifying who had more/less penguins. However, some pairs did experience difficulties in answering the question "How many more?" correctly. Some children appeared to answer somewhat randomly, without even looking at the representations at hand. To address this problem, Miss Smith always directed children's attention to the counting trays ("Look please, you need to look at the penguins first!"). She instructed the children to put the trays side by side (Figure 8) and count only those penguins who did not have a partner. Another strategy she suggested was to cover with a hand all the penguins with a partner, and then see how many were left alone.

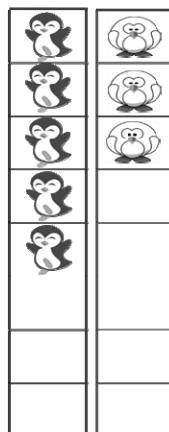


Figure 8. Diagram Showing Trays Side by Side

At some point of the lesson, Miss Smith started to reformulate the question from “How many more penguins?” to “How many penguins don’t have a friend?” As shown in the following transcript, this strategy helped children to derive the right answers.

Miss Smith: Look. This blue penguin has got a white penguin friend, right?

How about this one? Does this have a friend?

Mark: Yes!

Miss Smith: And how about this one? Does this have also a friend?

[points and matches the two adjacent penguins on both trays]

Mark: Yes.

Miss Smith: And this one? Does this have a friend?

Mark: No.

Miss Smith: And this?

Mark: Also no!

Miss Smith: So how many don’t have white friends?

Mark: One, two, three. Three with no white friends!

[counts with his fingers]

Station B: Dot cards

The materials provided in this working station were two sets of dot cards, similar to the ones shown in Figure 7. Each child was given a set of dot cards and placed with dots facing down. Both children had to draw one card at a time, determine the number of dots on their respective cards (by subitizing or counting), and determine who had more dots and how many more. Within this working station, Miss Smith was observed asking and repeating questions such as “How many dots are there?”, “Who has more/less dots?”, “Which one is a bigger number?”, and “How many more?” As in *Station A*, children only showed difficulties with the question “How many more?” In these cases, the teacher suggested children pay close attention to the cards and count only those dots that did not have a partner on the other card.

Station C: Cards with pictures, numerals, and number words

Children were given a stack of A3 picture cards with a row of green bears and another

row of yellow bears (Figure 9). Additionally, they were given a stack of smaller cards with numerals and number words (Figure 10). First, children were to pick one A3 picture card and count the number of bears in each row. Second, they had to identify the row with more bears (yellow or green), and determine how many more yellow/green bears there were. Finally, they had to pick the numeral or number word cards that corresponded to the differential number. Miss Smith was observed asking and repeating similar questions as the ones described above (e.g., “Who has more”, “How many more?”). Children only showed difficulties when determining the difference in the number of yellow and green bears. The strategies employed by the teacher to help children were similar to those used in *Station A*.



Figure 9. A3 Picture Cards Including Rows of Yellow and Green Bears

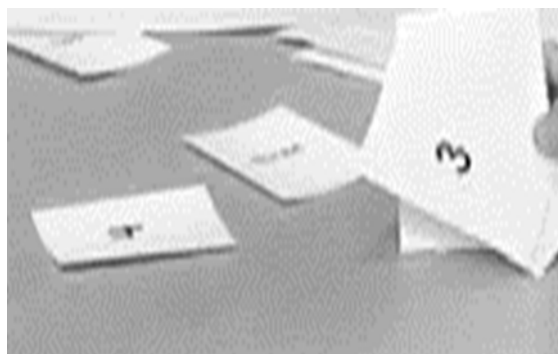


Figure 10. Numeral and Number Word Cards

Discussion

Few observational studies have explored how preschool educators may utilize multiple representations to teach numeracy to young children in the classroom (Braswell, 2015; Delacour, 2016). This study addresses this gap by analyzing a sample case from a Singapore K1 classroom. The teacher, Miss Smith, implemented a lesson that involved multiple ways of representing numbers. Our discussion below centers primarily on the activities and pedagogical strategies enacted by the teacher during the lesson.

To help 4- to 5-year-old children meaningfully learn about the concept of more-or-less and about the process of determining the difference between two quantities, Miss Smith employed a variety of representations: concrete (dice, penguin counters), pictorial (dot cards, rows of bears on picture cards), embodied (gestures, matching tally marks with the finger, indicating numbers with hands), verbal/written (counting out-loud, number word cards), tabular (table with two columns of tally marks), and symbolic representations (numerals, tally marks on paper, number words on cards) (Bautista, Roth, & Thom, 2011; Dreher et al., 2016; Sinclair & Pimm, 2015). This representational variety allowed children to manipulate and visualize the same numeracy notions (e.g., quantity, more than, less than, equal to) in concrete, pictorial, and abstract ways. This is consistent with Singapore's NEL framework for *Numeracy* (MOE, 2013b), which specifically encourages teachers to plan learning experiences based on the CPA approach. Note that children had been already exposed to all those representations in prior lessons. Likely due to this reason, Miss Smith did not see the need to design the activities according to the standard sequence (C→P→A) (Leong et al., 2015).

Miss Smith designed hands-on learning activities, which demanded children's physical engagement with the representations provided. Activities were consistent with recommendations by researchers in mathematical embodiment, who claim that bodily experiences and physical actions should always be involved in the learning of mathematical concepts and processes (Thom, Roth, & Bautista, 2010). Children could look, manipulate, make gestures, count, and compare quantities in objects such as penguin counters, cards with dots and bears, and a die. Some children even had the opportunity to act as concrete representations themselves when they stood within the hula-hoops for the other children to

count and compare. This was also aligned with proposals by constructivist and socio-cultural researchers, who claim that children construct mathematical knowledge by engaging in purposeful explorations of their physical and social environments (Braswell, 2015; Kamii et al., 2001).

The whole-class activities and pair working stations allowed children to gain experiences with multiple instances of more-or-less and forge connections among the different representations available (Siegler & Ramani, 2008). For example, this was clearly observed when children were asked to link information from picture cards with numerals and/or number words. They were able to translate information from one format to another successfully. Similar to Sabrina, the teacher featured in Jao's (2009) study, Miss Smith provided scaffolding for children to forge such connections. She paid close attention to children's responses and consistently acknowledged their ideas. Based on this, she posed follow-up prompts and questions, both open and close-ended, formulating them with different words to maximize children's understanding (e.g., "Which one has more?", "Which one is a larger number?"). Occasionally, she modeled and guided step-by-step how to read or interpret the representation/s at hand (e.g., how to count one by one, using a finger). Furthermore, she persisted in her efforts to provide constructive feedback to children. Note that these pedagogical strategies resembled the *explicit* and *implicit* mediation strategies reported by Braswell (2015).

Answering questions such as "Who has more?" was easy for all children in the classroom. However, the question "How many more?" was challenging for some of them. To help those children overcome their difficulties, Miss Smith reformulated the initial question into "How many are left without a partner?" Children were able to relate this new question more easily to the representations given. She also suggested children to *look carefully* at the representations before replying with random numbers, prompting them to focus attention on something tangible, and then taught them how to use their bodies (e.g., fingers, hands, eyes) to find out how many bears, dots, or penguins did not have a partner. In summary, instead of teaching about more-or-less in an abstract manner, Miss Smith provided tools for children to make their thinking concrete and externally available for reflection and communication (Jao, 2009).

Conclusion

We do not claim that these are new or innovative strategies and pedagogies for the teaching of numeracy in preschool education settings. As seen in the *Literature Review*, we are aware that many educational psychologists (Bruner, 1966), mathematics education researchers (Goldin & Kaput, 1996; Kamii et al., 2001), and curriculum designers (MOE, 2013b) have been advocating for this way of teaching for decades. Similarly, we do not mean to imply that Miss Smith carried out a ‘perfect’ lesson, which should be literally replicated by others. We agree with Coffield and Edward (2009) in that the idea of spreading ‘best’, ‘exemplary’, or ‘excellent’ practices is problematic and unrealistic in educational settings. We simply argue that the design features of the activities implemented by Miss Smith, alongside her pedagogical ‘moves’ in the classroom, led to a dynamic use of mathematical representations that might have resulted in meaningful numeracy learning among her young learners. We did not collect data on children’s learning as a result of this particular lesson; hence, this statement is simply a conjecture based on our observations of their actions and behaviors during the class.

The present study is significant because it contradicts the views and beliefs of many teachers who see representations as a *topic* of study reserved for high-achieving students rather than as a *tool* to foster deep and meaningful mathematical learning among all learners (Dreher & Kuntze, 2015; Dreher et al., 2016; Stylianou, 2010). Consistent with prior classroom-based studies conducted with young children (Braswell, 2015; Carruthers & Worthington, 2005, 2008; Delacour, 2016; MacDonald, 2013), Miss Smith’s lesson shows that it is possible to adopt a multiple representation teaching approach with young children (4-5 years old), and that children can actively and productively engage in learning activities where foundational numeracy concepts (such as more-or-less) are taught using different representational forms. Moreover, we have discussed certain pedagogical strategies employed by Miss Smith while conducting her lesson, including employing a variety of representations, forging meaningful connections among them, encouraging children’s physical involvement, scaffolding children’s learning through prompts, questions, and modelling, and adjusting instructional strategies based on children’s needs (Braswell, 2015; Delacour, 2016; Jao, 2009).

Future Research and Implications

Future case studies should be conducted within the same educational level to document other design features and pedagogical strategies not observed in Miss Smith's lesson. It would also be important to follow-up with studies looking into children's numeracy learning outcomes as a result of multiple representations teaching approaches, comparing with other teaching approaches where representations play a secondary role.

Because important discrepancies seem to exist between curriculum frameworks and practices on the ground, resources are needed to strengthen preschool teachers' pedagogical approaches to early mathematics (Alsina, 2013; Moon & Lee, 2011). The intent of this qualitative study has been to increase awareness among preschool educators about the important role representations play in numeracy learning, and provide insights that inspire and encourage them to incorporate multiple representations in their classroom activities. Thus, we invite preschool teachers to reflect on how the sample case analyzed here might inform their own numeracy-related instructional practices.

Acknowledgements

This study was funded by Singapore Ministry of Education (MOE) under the Education Research Funding Programme (OER 09/14RB) and administered by National Institute of Education (NIE), Nanyang Technological University, Singapore. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Singapore MOE and NIE.

References

- Alsina, A. (2013). Early childhood mathematics education: Research, curriculum, and educational practice. *REDIMAT: Journal of Research in Mathematics Education*, 2(1), 100-153. doi:104471/redimat.2013.22
- Bautista, A., Cañadas, M. C., Brizuela, B. M., & Schliemann, A. (2015). Examining how

- teachers use graphs to teach mathematics in a professional development program. *Journal of Education and Training Studies*, 3(2), 91-106. doi:10.11114/jets.v3i2.676
- Bautista, A., Roth, W. M., & Thom, J. S. (2011). Knowing, learning, and the integrity of kinetic movement. *Interchange: A Quarterly Review of Education*, 42(2), 363-388. doi:10.1007/s10780-012-9164-9
- Braswell, G. S. (2015). Observations of representational practices by Indian-descent children in a US preschool classroom: Connections among people, spaces and artifacts. *Early Childhood Education Journal*, 43(2), 135-142. doi:10.1007/s10643-014-0637-6
- Brizuela, B. M., & Gravel, B. (2013). *Show me what you know: Exploring representations across STEM disciplines*. New York, NY: Teachers College Press.
- Bruner, J. S. (1966). *Toward a theory of instruction*. London, England: Harvard University Press.
- Bull, R., & Bautista, A. (2018). A careful balancing act: Evolving and harmonizing a hybrid system of ECEC in Singapore. In S. L. Kagan (Ed.), *The early advantage: Early childhood systems that lead by example* (pp. 155-181). New York, NY: Teachers College Press.
- Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, 105(2), 380-400. doi:10.1037/a0031084
- Carruthers, E., & Worthington, M. (2005). Making sense of mathematical graphics: The development of understanding abstract symbolism. *European Early Childhood Education Research Journal*, 13(1), 57-79. doi:10.1080/13502930585209561
- Carruthers, E., & Worthington, M. (2008). Children's mathematical graphics: Young children calculating for meaning. In I. Thompson (Ed.), *Teaching and learning early number* (pp. 127-148). Maidenhead, England: Open University Press.
- Coffield, F., & Edward, S. (2009). Rolling out 'good', 'best' and 'excellent' practice. What next? Perfect practice? *British Educational Research Journal*, 35(3), 371-390. doi:10.1080/01411920802044396
- Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

- Darling-Hammond, L., & McLaughlin, M. W. (2011). Policies that support professional development in an era of reform. *Phi Delta Kappan*, 92(6), 81-92. doi:10.1177/003172171109200622
- Delacour, L. (2016). Mathematics and didactic contract in Swedish preschools. *European Early Childhood Education Research Journal*, 24(2), 215-228. doi:10.1080/1350293X.2016.1143257
- Dreher, A., & Kuntze, S. (2015). Teachers' professional knowledge and noticing: The case of multiple representations in the mathematics classroom. *Educational Studies in Mathematics*, 88(1), 89-114. doi:10.1007/s10649-014-9577-8
- Dreher, A., Kuntze, S., & Lerman, S. (2016). Why use multiple representations in the mathematics classroom? Views of English and German preservice teachers. *International Journal of Science and Mathematics Education*, 14(2), 363-382. doi:10.1007/s10763-015-9633-6
- Goldin, G., & Kaput, J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 397-430). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Goldin, G., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. A. Cuoco, & F. R. Curcio (Eds.), *The roles of representation in school mathematics: 2001 yearbook of the national council of teachers of mathematics* (pp. 1-23). Reston, VA: NCTM.
- Gürbüz, R. (2010). The effect of activity-based instruction on conceptual development of seventh grade students in probability. *International Journal of Mathematical Education in Science and Technology*, 41(6), 743-767. doi:10.1080/00207391003675158
- Jao, L. (2009). *Constructing mathematical knowledge using multiple representations: A case study of a grade one teacher* (Unpublished doctoral dissertation). University of Toronto, Toronto, Canada.
- Kamii, C., Lewis, B. A., & Kirkland, L. (2001). Manipulatives: When are they useful? *The Journal of Mathematical Behavior*, 20(1), 21-31. doi:10.1016/S0732-3123(01)00059-1
- Leong, Y. H., Ho, W. K., & Cheng, L. P. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator*, 16(1), 1-18.

- MacDonald, A. (2013). Using children's representations to investigate meaning-making in mathematics. *Australasian Journal of Early Childhood*, 38(2), 65-73. doi:10.1177/183693911303800209
- Ministry of Education (MOE). (2013a). *Nurturing early learners: A curriculum for kindergartens in Singapore. Discovery of the world*. Singapore, Republic of Singapore: Ministry of Education.
- Ministry of Education (MOE). (2013b). *Nurturing early learners: A curriculum for kindergartens in Singapore. Numeracy*. Singapore, Republic of Singapore: Ministry of Education.
- Ministry of Education and Human Resources Development. (2007). *The kindergarten curriculum of the Republic of Korea*. Retrieved from <http://ncm.gu.se/media/kursplaner/andralander/koreaforskola.pdf>
- Moon, Y. S., & Lee, H. Y. (2011). A study on the early childhood mathematics teachers' perceptions and practices in support of the goals and contents of early childhood mathematics education. *Asia-Pacific Journal of Research in Early Childhood Education*, 5(2), 67-92.
- Puchner, L., Taylor, A., O'Donnell, B., & Fick, K. (2008). Teacher learning and mathematics manipulatives: A collective case study about teacher use of manipulatives in elementary and middle school mathematics lessons. *School Science and Mathematics*, 108(7), 313-325.
- Sandelowski, M. (1995). Sample size in qualitative research. *Research in Nursing and Health*, 18(2), 179-183. doi:10.1002/nur.4770180211
- Sherman, J., & Bisanz, J. (2009). Equivalence in symbolic and nonsymbolic contexts: Benefits of solving problems with manipulatives. *Journal of Educational Psychology*, 101(1), 88-100.
- Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science*, 11(5), 655-661. doi:10.1111/j.1467-7687.2008.00714.x
- Sinclair, N., & Pimm, D. (2015). Mathematics using multiple senses: Developing finger gnosis with three- and four-year-olds in an era of multi-touch technologies. *Asia-Pacific Journal of Research in Early Childhood Education*, 9(3), 99-110.

doi:10.17206/apjrece.2015.9.3.99

Stylianou, D. A. (2010). Teachers' conceptions of representation in middle school mathematics. *Journal of Mathematics Teacher Education*, 13(4), 325-343.

doi:10.1007/s10857-010-9143-y

Thom, J. S., Roth, W. M., & Bautista, A. (2010). In the flesh: Living, growing conceptual domains in a geometry lesson. *Complicity: An International Journal of Complexity and Education*, 7(1), 77-87.